# Hornady 4 Degree of Freedom (4 DOF) Trajectory Program 

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This document includes multiple sections to provide information and background on the use of the Hornady 4 Degree of Freedom (DOF) Trajectory Program. The User manual explains the background and capability of the Hornady 4 DOF. The Important Factors and Tips provides information regarding the use of the program. The Additional Information section contains detailed information on various subjects pertaining to the understanding and application of the Hornady 4 DOF outputs. We sincerely hope you find the new Hornady 4 DOF Trajectory Program accurate and useful.

## Introduction

The Hornady 4 DOF trajectory program is a state of the art trajectory engine that utilizes a modified point mass solution to provide incredibly accurate trajectories for listed projectiles to extremely long ranges. Traditional Siacci based BC codes do not consider projectile dynamic flight characteristics and their contributions to the final trajectory solution. The Hornady 4 DOF accurately models the dynamic characteristics which can effect and modify the projectile's trajectory. The software does not use Ballistic Coefficient (BC), but instead utilizes Drag Coefficient (Cd) versus Mach number for each projectile. To use the program you will select a specific projectile from a pull down menu instead of inputting a BC . When a projectile is selected, the program inputs projectile mass properties, aerodynamic moments, and coefficients to include Doppler radar determined drag coefficients specific to each projectile. The software accurately predicts drop, wind drift, projectile Gyroscopic Stability Factor ( Sg ) as a function of range, yaw of repose and corresponding prediction of spin drift, aerodynamic jump due to a cross wind, and limit cycle yaw at extended ranges due to Magnus effects.

All listed projectiles have been extensively tested with Doppler radar during development and analysis of the program accuracy. Output values have been compared to Doppler radar data at ranges as far as 2,000 yards with predicted errors being within single digits of radar data for retained velocity. It is not possible to obtain this level of fidelity to actual real world data any other way. The use of BC is a good approximation of trajectories, but becomes increasingly inaccurate at various points during the trajectory. For this reason truing the BC to reflect actual retained velocity or drop data has been the common method used to address the errors in prediction when using $B C$. The inaccuracies seen when using $B C$, even $G 7$, are due to the mismatch between the actual drag of the projectile being fired and the drag of the standard projectile being used to model it. As projectiles are shot to longer and longer ranges, trajectory predictions based on $B C$ become increasingly inaccurate in elevation, become substantially inaccurate in wind and spin drift, and offer no prediction for aerodynamic jump due to crosswind.

The superiority of Cd over $\mathrm{G7}$ can be shown in a comparison of the Cd versus Mach number of the G7 standard projectile to that of several currently produced projectiles Figure 1. As can be seen, other than the 220 gr ELD-X ${ }^{\text {TM }}$ none of the other projectiles match up with the G 7 drag curve in both Cd value and curve shape. This will inevitably lead to errors in trajectory predictions at longer ranges and especially
at certain points in the trajectory where drag curves do not match up. The only way to truly and accurately model the trajectory of a projectile is with specific mass and aerodynamic models, specific drag data and modeling of the dynamic behavior of the projectile.


Figure 1: Cd vs. Mach number For Various Projectiles
The G7 Ballistic coefficient does a better job of predicting the trajectory of modern long ogive, boat tail bullets to longer ranges than the G1 standard, however, it is still not modeling the exact Cd value or shape and will result in errors. Utilizing the popular and well-designed JBM ballistics code, Table 1. shows a comparison of downrange predictions between G7 and Hornady 4 DOF. A Hornady 6.5 mm 140 ELD-Match projectile is compared for retained velocity, drop, and spin drift. Table 2. shows the same comparison of wind drift and drop values with aerodynamic jump due to a crosswind.

| RETAINED VELOCITY |  |  |  | DROP INCHES |  |  |  | SPIN DRIFT INCHES |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| YDS | JBM G7 | 4 DOF | DIFF | YDS | JBM G7 | 4 DOF | DIFF | YDS | JBM G7 | 4 DOF | DIFF |
| 0 | 2785 | 2785 | 0 | 0 | -2.3 | -2.3 | 0.0 | 0 | 0.0 | 0.0 | 0.0 |
| 100 | 2638 | 2645 | -7 | 100 | 0.0 | 0.0 | 0.0 | 100 | 0.1 | 0.0 | -0.1 |
| 200 | 2494 | 2510 | -16 | 200 | -2.7 | -2.7 | 0.0 | 200 | 0.3 | 0.2 | -0.2 |
| 300 | 2356 | 2379 | -23 | 300 | -11.0 | -10.9 | 0.2 | 300 | 0.6 | 0.4 | -0.2 |
| 400 | 2221 | 2251 | -30 | 400 | -25.6 | -25.2 | 0.4 | 400 | 1.0 | 0.7 | -0.3 |
| 500 | 2092 | 2126 | -34 | 500 | -47.2 | -46.4 | 0.8 | 500 | 1.6 | 1.1 | -0.5 |
| 600 | 1967 | 2003 | -36 | 600 | -76.7 | -75.3 | 1.5 | 600 | 2.4 | 1.6 | -0.8 |
| 700 | 1846 | 1882 | -36 | 700 | -115.3 | -112.8 | 2.5 | 700 | 3.4 | 2.3 | -1.1 |
| 800 | 1728 | 1761 | -33 | 800 | -164.1 | -160.2 | 3.9 | 800 | 4.6 | 3.1 | -1.5 |
| 900 | 1614 | 1640 | -26 | 900 | -224.5 | -218.8 | 5.7 | 900 | 6.1 | 4.2 | -1.9 |
| 1000 | 1503 | 1520 | -17 | 1000 | -298.4 | -290.3 | 8.1 | 1000 | 7.9 | 5.4 | -2.5 |
| 1100 | 1395 | 1400 | -5 | 1100 | -387.6 | -376.9 | 10.7 | 1100 | 10.0 | 6.8 | -3.2 |
| 1200 | 1292 | 1282 | 10 | 1200 | -494.8 | -481.3 | 13.5 | 1200 | 12.6 | 8.5 | -4.1 |
| 1300 | 1194 | 1166 | 28 | 1300 | -622.8 | -606.8 | 16.0 | 1300 | 15.7 | 10.4 | -5.3 |
| 1400 | 1103 | 1066 | 37 | 1400 | -775.4 | -758.0 | 17.4 | 1400 | 19.3 | 12.7 | -6.6 |
| 1500 | 1048 | 1009 | 39 | 1500 | -956.4 | -939.5 | 16.9 | 1500 | 23.7 | 15.3 | -8.4 |
| 1600 | 1014 | 975 | 39 | 1600 | -1169.1 | -1154.9 | 14.2 | 1600 | 28.6 | 18.4 | -10.2 |
| 1700 | 985 | 949 | 36 | 1700 | -1415.7 | -1406.8 | 8.9 | 1700 | 34.1 | 21.9 | -12.2 |
| 1800 | 959 | 923 | 36 | 1800 | -1698.4 | -1697.3 | 1.1 | 1800 | 40.3 | 25.8 | -14.5 |
| 1900 | 935 | 897 | 38 | 1900 | -2019.0 | -2028.4 | -9.4 | 1900 | 47.0 | 30.3 | -16.7 |
| 2000 | 913 | 871 | 42 | 2000 | -2379.8 | -2402.7 | -22.9 | 2000 | 54.5 | 35.4 | -19.2 |

Table 1. Trajectory Predictions for Hornady 6.5 mm 140 gr ELD ${ }^{\text {m }}$ Match

| WIND 10 MPH 90* |  |  |  | DROP WIND @ 10 MPH 90* |  |  |  | DROP WIND @ 10 MPH 270* |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| YDS | JBM G7 | 4 DOF | DIFF | YDS | JBM G7 | 4 DOF | DIFF | YDS | JBM G7 | 4 DOF | DIFF |
| 0 | 0.0 | 0.0 | 0.0 | 0 | -2.3 | -2.3 | 0.0 | 0 | -2.3 | -2.3 | 0.0 |
| 100 | 0.5 | 0.5 | 0.0 | 100 | 0.0 | 0.3 | 0.3 | 100 | 0.0 | -0.3 | -0.3 |
| 200 | 2.1 | 2.0 | 0.1 | 200 | -2.7 | -2.1 | 0.7 | 200 | -2.7 | -3.3 | -0.6 |
| 300 | 5.0 | 4.7 | 0.3 | 300 | -11.0 | -10.0 | 1.1 | 300 | -11.0 | -11.8 | -0.8 |
| 400 | 9.1 | 8.5 | 0.6 | 400 | -25.6 | -24.0 | 1.6 | 400 | -25.6 | -26.4 | -0.8 |
| 500 | 14.6 | 13.7 | 0.9 | 500 | -47.2 | -44.9 | 2.3 | 500 | -47.2 | -47.9 | -0.7 |
| 600 | 21.7 | 20.3 | 1.4 | 600 | -76.7 | -73.5 | 3.2 | 600 | -76.7 | -77.0 | -0.3 |
| 700 | 30.5 | 28.6 | 2.0 | 700 | -115.3 | -110.7 | 4.6 | 700 | -115.3 | -114.9 | 0.4 |
| 800 | 41.1 | 38.6 | 2.5 | 800 | -164.1 | -157.8 | 6.3 | 800 | -164.1 | -162.6 | 1.6 |
| 900 | 53.7 | 50.7 | 3.0 | 900 | -224.5 | -216.1 | 8.4 | 900 | -224.5 | -221.4 | 3.1 |
| 1000 | 68.7 | 65.2 | 3.5 | 1000 | -298.4 | -287.3 | 11.1 | 1000 | -298.4 | -293.3 | 5.1 |
| 1100 | 86.2 | 82.4 | 3.8 | 1100 | -387.6 | -373.6 | 14.0 | 1100 | -387.6 | -380.2 | 7.5 |
| 1200 | 106.6 | 102.9 | 3.7 | 1200 | -494.8 | -477.7 | 17.1 | 1200 | -494.8 | -484.8 | 10.0 |
| 1300 | 130.2 | 127.1 | 3.1 | 1300 | -622.8 | -602.9 | 19.9 | 1300 | -622.8 | -610.7 | 12.1 |
| 1400 | 157.3 | 155.7 | 1.6 | 1400 | -775.4 | -753.8 | 21.6 | 1400 | -775.4 | -762.1 | 13.3 |
| 1500 | 187.7 | 187.8 | -0.1 | 1500 | -956.4 | -935.0 | 21.4 | 1500 | -956.4 | -943.9 | 12.5 |
| 1600 | 220.1 | 222.2 | -2.1 | 1600 | -1169.1 | -1150.2 | 18.9 | 1600 | -1169.1 | -1159.7 | 9.4 |
| 1700 | 254.1 | 258.2 | -4.1 | 1700 | -1415.7 | -1401.8 | 13.9 | 1700 | -1415.7 | -1411.9 | 3.8 |
| 1800 | 289.6 | 295.8 | -6.2 | 1800 | -1698.4 | -1692.0 | 6.5 | 1800 | -1698.4 | -1702.6 | -4.2 |
| 1900 | 326.6 | 335.1 | -8.5 | 1900 | -2019.0 | -2022.8 | -3.8 | 1900 | -2019.0 | -2034.0 | -15.0 |
| 2000 | 365.1 | 376.1 | -11.0 | 2000 | -2379.8 | -2396.8 | -17.0 | 2000 | -2379.8 | -2408.6 | -28.8 |

Table 2. Trajectory Predictions for Hornady 6.5 mm 140 gr ELD ${ }^{\text {TM }}$ Match

## Gyroscopic Stability

Gyroscopic stability $(\mathrm{Sg})$ is merely a measure of the projectile's ability to maintain point forward flight and not assume a large and variable angle of attack. A minimum gyroscopic stability factor at the muzzle of 1.0 is required for a bullet to fly point first. Sg depends on the projectile's mass, moments of inertia, spin rate, air density, pitching moment and velocity. Existing stability calculators available today based off bullet length are a good rule of thumb estimate, but they are exactly that, approximations. Without properly modeling mass distribution inside the projectile as well as the effect its unique shape has on the location of the Normal Force Center of Pressure location, accurate gyroscopic stability calculations using the Greenhill or Miller stability calculations are estimates. The Hornady 4 DOF accurately calculates Sg based on each projectiles mass, aerodynamic properties, and atmospheric conditions. It must be pointed out that a projectile gets rapidly more stable gyroscopically as it flies downrange. The spin of a projectile decays at a much slower rate than its axial velocity does. The changing aerodynamic properties as the projectile slows, without the spin appreciably changing, results in a more and more stable projectile as it flies downrange.

In general, a projectile with Sg values at the muzzle, in ambient atmospheric conditions, of around 1.4 is considered the lower limit. This allows for some error in the calculation of the Pitching Moment and for increased air density when the projectile is fired under cold conditions at low altitudes. Hornady 4 DOF will display the Sg of the bullet as it flies downrange. If you have an Sg of less than 1.4 at the muzzle under ambient conditions we would recommend you model a faster twist rate, and consider using a faster twist barrel. Extensive Doppler radar testing has shown that for supersonic Mach numbers above 1.7-1.8 that the drag of a projectile reaches a minimum at an Sg of about 2.0. The transonic drag on a projectile will continue to decrease as the spin rate and Sg is increased. There are practical limits of this as you can spin a projectile to the point that it will mechanically fail in flight from excessive centrifugal force. Excessively spinning a non-expanding bullet will have detrimental effects on its terminal performance.

After running a trajectory, the first check should be the highlighted "Gyro" column of the outputs table, see Figure 2. The Hornady 4 DOF will not produce an error if the Sg is below 1.0 at the muzzle. Instead, the 4 DOF will model the projectile as unstable until it has lost enough velocity to climb back to a Sg of 1.0. When launched with a Sg of less than 1.0, velocity loss occurs extremely rapidly and should appear abnormal. Figure 3. is an example output table showing the highlighted area that should be checked by the user to ensure a properly stabilized bullet. This value should be checked for each trajectory ran.

## Trajectory Results

| RANGE (YDS) | TOTAL COME UP (IN) | total WINDAGE <br> (IN) | TRAJECTORY <br> (IN) | AERODYNAMIC JUMP (IN) | WIND DRIFT <br> (IN) | SPIN DRIFT (IN) | VELOCITY <br> (FPS) | $\begin{aligned} & \text { ENERGY } \\ & \text { (FT-LB) } \end{aligned}$ | TOF (SEC) | GYRO |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | +1.5 | 0 | -1.50 | -0.00 | 0.00 | 0 | 2600 | 1577 | 0.000 | 2.08 |

Figure 2. Sample Output

## Zero Angle

Version II of 4 DOF $^{\text {TM }}$ has a unique and very useful feature added called Zero Angle. Zero Angle is the angle of the bore of the rifle relative to the Line of Sight (LOS) of the optic. It is dependent on the geometry of the scope mounting relative to the rifle bore and the scope adjustment setting or zero. Any changes in environmental conditions from zeroing to actual field shooting can have an effect on the actual zero range crossing of the projectile and the line of sight. This includes changes in temperature, pressure / altitude, humidity, wind speed, and wind angle. If the changes these environmental effects have on the actual zero range are not accounted for, misses to varying degrees are likely.

Zero Angle does not change with drastic changes in atmospheric conditions like a Zero Range can. As long as the scope zero is not changed the mechanical relationship between the bore angle and line of sight will remain the same. Once the Zero Angle is determined for a given load, atmospheric condition and Zero Range it can be considered a fixed value. The program now allows the user to enter a Zero Range, which can be dependent on atmospheric conditions, or the Zero Angle which is independent of atmospheric conditions. This allows the user to utilize a Zero Angle instead of Zero Range and eliminate any errors associated with a change in atmospheric conditions effecting the actual Zero Range and corresponding trajectory. Using Zero Angle allows the user to go to any atmospheric condition, no matter how different from their zeroing conditions, and the 4DOF ${ }^{\text {TM }}$ will output the correct zero distance and trajectory without having to re-zero the rifle to those specific conditions.

To determine the Zero Angle specific to your rifle and load, follow the recommended zeroing procedures described later in this document, or in video form on the Hornady Youtube page. The 4DOF will output both a Zero Range and Zero Angle for every trajectory that is ran in the Your Input Variables portion of the Trajectory Results Table. After following the recommended zeroing procedures, the user should note the Zero Angle for future use. For all following trajectory calculations, the user can select Zero Angle on the input page instead of using a traditional Zero Range. Using Zero Angle ensures that any variance in atmospheric conditions will not inadvertently effect trajectory calculations using the 4DOF.

## Muzzle Velocity Correction

For precise shooting, muzzle velocity needs to be very accurately known. All propellants change performance as a function of temperature and thus the pressure and muzzle velocity change as well. Some propellants change much less than others. Typically single base, no Nitroglycerin (NG), propellants change the least followed by double base propellants, which incorporate NG and the biggest variation is usually found with BALL™ propellants. Depending on caliber, primer, and especially the propellant, loads can change by as little as $30-50$ feet per second ( fps ) over a 150 degrees F temperature differential to as much as hundreds of fps. To make a long story short, the most temperature stable propellants as of the writing of this paper, November 2016, are the Hodgdon Extreme Series and the new IMR Enduron ${ }^{\text {M }}$
propellants. To account for these possible changes in muzzle velocity due to temperature Version II of 4DOF incorporates a Temperature Sensitivity Coefficient (TSC) feature.

## Custom Temperature Sensitivity Coefficient

To achieve the most accurate muzzle velocity prediction, the user should test their load at two widely different temperatures as well as a baseline temperature and accurately record the velocities. An example temperature range would be 20 deg for below baseline, 70 deg for baseline, and 100 deg for above baseline. The temperature range of the environment you will be shooting in should dictate your tested temperatures. When conducting testing, it is important to make sure the ammunition has enough soak time to reach the actual temperature being tested. Velocity and temperature recordings can be entered into The Temperature Sensitivity Coefficient Calculator to determine the custom TSC's that will be entered into the 4DOF. Manual calculation can be conducted by dividing the difference in velocity by the difference in temperature between two test temperatures.

Once the TSC's are calculated and entered into the 4DOF the baseline temperature and atmospheric temperatures must be input and will be used to calculate the change in velocity. The input muzzle velocity at the baseline temperature will be automatically altered based on the TSC, the baseline temperature, and the atmospheric temperature once the trajectory is calculated. The altered velocity can be viewed in the upper left portion of the "Your Input Variables" of the "Trajectory Results" page.

Most propellants show a lower muzzle velocity at cold temperatures and a higher velocity at hot temperatures when compared to the baseline temperature velocity. The amount of change in velocity per degree change in temperature usually is not exactly the same for above baseline temperatures vs below baseline temperatures and therefore will result in a unique TSC for temperatures above and below the baseline. In this case the user will need to enter the correct TSC based on the atmospheric temperature being above or below their baseline temperature. In the rare case that velocities rise at temperatures below baseline or velocity drops at temperatures above baseline, a negative TSC value should be entered.

## General Temperature Sensitivity Coefficient

If the user is unable to conduct their own testing, Version II of 4DOF also allows the user to choose a propellant and associated TSC from a pull down menu. The baseline temperature when recording muzzle velocity is still required to accurately calculate changes in velocity. After inputting the baseline temperature, the powder being used should be selected from the drop down menu. A general TSC will automatically be used based on the selected powder, baseline temperature, and atmospheric temperature to alter the muzzle velocity.

The user determined TSC will provide more accurate results than the table, which is an average TSC of the selected propellant tested by Hornady in a number of different loads and with different components. If the custom TSC is not available, utilizing the provided table of TSC's will provide more accurate results than not accounting for temperature effects on muzzle velocity at all.

If the user does not want to use the TSC feature of the 4DOF, simply leave the Temperature Sensitivity Coefficient box unchecked and the 4DOF will operate based on a simple muzzle velocity.

Note: We have not included any BALL™ propellants in the TSC table because they typically have much greater temperature sensitivity as compared to most other propellants. Because of this we do not consider them appropriate for use in long range, high precision shooting applications. If you are using BALL Propellants we highly recommend you use the Custom TSC application in 4DOF and determine the TSC for your load.

## Spin Drift, Drift Due to the Yaw of Repose

As a projectile flies down range, the trajectory is curved in the vertical plane by the action of gravity. The trajectory curvature in the vertical plane, combined with the gyroscopic moment arising from the projectile spin and inertia, make the projectile nose point slightly up relative to the velocity vector of the bullet. The projectile nose pointing up relative to its velocity vector, combined with the trajectory curvature induces an angular rate on the projectile. If the bullet spin is in the "right hand" sense (spin axis aligned with your thumb, the direction of rotation aligned with your fingers), the projectile angular momentum makes the bullet point nose right relative to its velocity vector. The projectile nose pointing slightly to the right causes higher pressure on the left side of the projectile. This pressure differential left to right makes the bullet "drift" to the right for right hand twist barrels. The drift is known as the "drift due to the yaw of repose" or "spin drift". The drift due to yaw of repose depends on the ballistic drop of the bullet, the twist of the barrel, and the inertial and aerodynamic characteristics of the projectile.

The Hornady 4 DOF trajectory code correctly calculates the drift due to yaw of repose for each bullet/twist/velocity combination, something that the old "Siacci" based trajectory programs can only estimate.

## Aerodynamic Jump

Aerodynamic jump is an interesting aerodynamic phenomenon that occurs when spin stabilized projectiles are fired into cross winds. The gyroscopic spin moment of the projectile causes the bullet nose to suddenly point up or down depending on the cross wind direction, making the projectile jump in the vertical direction. This phenomenon is known as "aerodynamic jump" and arises because of the interaction of gyroscopic moment of the projectile and angular rate induced when the projectile suddenly enters a cross flow from winds acting perpendicular to the muzzle velocity vector. If the wind is blowing from right to left, the aerodynamic jump is "up" in the vertical direction. When firing in cross winds from the left, the vertical jump is down. The aerodynamic jump is a fixed angle that depends on the magnitude of the wind, the twist of the barrel, and the inertial and aerodynamic characteristics of the projectile. A more detailed explanation of aerodynamic jump can be found in the Additional Information section below.

The Hornady 4 DOF trajectory code correctly calculates the aerodynamic jump for each bullet/twist/velocity combination, something that the old "Siacci" based trajectory programs can't do.

## Earth Based Effects

Earth based effects are commonly referred to as "Coriolis Effect". The effect is actually what is known as the Eotvos Effect for those who wish to do further research. For our purposes here we will use "Earth Based Effects" (EBE) as a catch all. In Version II of 4DOF when EBE effects are turned on the program is calculating the trajectory of the projectile including both elevation and drift components caused by the projectile velocity vector relative to the earth as well as curved earth effects.

Many assume that the EBE is due to the earth rotating under the projectile. This is in fact not true. For all practical purposes, in small caliber ballistics, the projectile remains attached to the reference frame of the earth. Simply put, the projectile starts out with the rotational velocity of the earth at the point it is fired and keeps that velocity component throughout its flight. What is actually happening is the projectile's velocity is creating an increase or decrease in inertia, or a centrifugal force, which for a very brief time is trying to push the projectile either further away or closer to the earth.

One way to picture the EBE is to picture an earth satellite. The faster the satellite travels the higher it orbits relative to the center of the earth. If it attains $17,000 \mathrm{mph}$, relative to the earth, it will leave earth orbit. Rockets are typically fired from west to east to take advantage of the approximately $1,000 \mathrm{mph}$ rotational velocity at the equator, so as to carry less fuel and more payload. If a rocket is fired from east to west the rocket is now working against the $1,000 \mathrm{mph}$ rotational velocity and it must carry more fuel and less payload, in order to overcome the $1,000 \mathrm{mph}$ in the wrong direction, to achieve the same orbital height. In the same way, if a projectile is fired west to east, for a brief period it has a velocity adding to the earth's rotational velocity and will want to be at a greater distance from the center of the earth, higher than it started out at. And conversely, if it is fired from east to west it is now working against the earth's rotational velocity and wants to be closer to the center of the earth, lower than it started out at. Bear in mind this is a VERY simplified explanation of what is happening, but hopefully helps to explain a hard to visualize physical concept.

When firing at any north or south azimuths this adds a horizontal drift component to the projectile trajectory. Remember from above, the projectile has the same reference frame as the point on earth it was fired from. The projectile will have the rotational velocity vector at the point it was launched from. The earth has the highest surface rotational velocity at the equator. Because it is a spherical surface, as you move away from the equator, north or south, the rotational speed at the surface is slower because of the smaller radius. In the northern hemisphere if you fire north the projectile will have a west to east velocity component greater than the impact point because you are firing to a smaller radius point on the earth. The projectile will drift slightly to the right. In the northern hemisphere if you fire south you are firing to a point that is moving faster than the point the bullet left from and again the projectile will drift to the right of the target. In the southern hemisphere everything is the opposite, the projectile will drift to the left of the target. Bear in mind that at typical small arms ranges this drift is pretty small.

To accurately predict the earth based effects on the projectile you must know relatively accurately what latitude you are firing from and what azimuth (compass angle) you are firing to. Earth based effects can essentially be ignored out to ranges of 1,000 yards as they are very small. We have done extensive testing at 1,500 yards and have found the earth based effects to be equal to or less than the normal variations of muzzle velocity, drag variation from bullet to bullet and tail or head wind effects.

## Notes on the use of the "Axial Force Form Factor" Input to the Hornady 4DoF Trajectory Engine

The Hornady 4 DOF is only as accurate as the data feeding it, and Hornady has taken great pains to ensure the drag vs. Mach number data for each projectile model used in the new trajectory engine is based on an average of a very large data sample. Hornady has fired each of the projectiles in this database from several rifles and we have occasionally noticed some minor shifts in the drag vs. Mach number curves from barrel-to-barrel within the same sample of bullets. Any actual drag shift for a given barrel depends on the barrel geometry, twist rate, and quality of manufacture (internal roughness, straightness, etc.). Figure 3. shows an extreme example of observed drag variability of a large number of identical projectiles fired from different barrels with different twists and different rifling geometries. Doppler radar has also shown significant changes in the projectile drag as a function of the type of propellant used and the muzzle brake type.


Figure 3. Drag vs. Mach Number for Identical Projectiles and Different Barrels
Generally, the average drag vs. Mach number profile for a standard projectile fired from a given barrel will differ only by a small amount (a "bias" from nominal), usually a fixed percent shift from the baseline.

Hornady has fired each of the projectiles in the database through several barrels, and it's our opinion that when using the 4 DOF, you should use a default drag form factor of 1.00 to initially simulate the flight path of projectiles. This will provide the expected bullet path of the "average" bullet fired from an "average" barrel. Depending on the details of the barrel, load and muzzle brake you are using, bullets fired from your gun may have drag vs. Mach number behavior that is a small percentage higher or lower than the data we have provided. While there are random drag variations shot-to-shot on the order of $2 \%$,
the average vertical impact point at extended ranges is strongly influenced by things like the air density, average muzzle velocity, average projectile weight, as well as the average drag vs. Mach number profile.

If you have simulated the flight path of our projectiles, and closely monitored the mean point of impact at zero range along with muzzle velocity via an accurate chronograph, but find your impact points are different than the Hornady 4 DOF predicts, you can make a minor shift (a "bias") in the impact point in the vertical plane by making a minor adjustment to the projectile drag form factor. If the actual mean point of impact of the projectiles fired at extended ranges is high relative to the predicted impact point (drag is lower than what is being used and it is causing the program to predict too much elevation adjustment), incrementally reduce the drag form factor in the Hornady 4 DOF until the simulated trajectory path matches your vertical impact point testing at extended range. Conversely, if the average impact point of your groups is below the impact point predicted by the Hornady trajectory engine (drag is higher than what is being used and the program is under predicting elevation adjustment), incrementally increase the drag form factor until you have matched the observed vertical impact.

The use of the axial force form factor to true your specific rifle should only be used after eliminating other possible variables that can account for an error in point of impact. The user should check the list in Table 3. to ensure none of the listed variables are contributing to an observed vertical point of impact difference between live fire data and 4 DOF prediction.

| Incorrect Muzzle Velocity |
| :---: |
| Error in Zero Range |
| Error in Sight Height |
| Accounted for Aiming Error at Range |
| Error in Parallax Setting |
| Uncalibrated Turret Adjustments |
| Unaccounted for Wind Speed and Direction |
| Inaccurate Atmospheric Data |

Table 3. Point of Impact Error Accountability Checklist

The use of secondary instrumentation such as an additional chronograph, rangefinder, or weather station may be needed to validate accuracy of inputs. Once you have adjusted the drag vs. Mach profile to your specific rifle based off of a repeatable mean point of impact at distance, you should find it accurately predicts the bullet path all along the trajectory and in environmental conditions other than those tested in. There will be no need to adjust things for other conditions, 4DOF will automatically do this.

The data examined on projectiles fired from various barrels indicates that accurate simulation of the bullet path is usually within a drag form factor between 0.95 and 1.05 for just barrel differences. For maximum flexibility and to account for different loads and muzzle devices, the program allows for axial force form factor adjustments from .90-1.10. If you find that is not the case, be sure to check the factors in table 3.

This method of tweaking or adjusting the projectile drag is different than and much more accurate than the method of "truing" a BC. When a BC is trued, not only is it trying to account for drag variance from barrel to barrel, but also for the mismatch in shape of the drag curve of the standard projectile and the actual projectile being fired. In extreme cases this method of truing can produce Cd vs Mach values that match well at one speed of sound, but can produce errors when used in a drastically different speed of sound. In other words, the trued BC that gave good drop values at 80 degrees may not produce the same accuracy of outputs at 20 degrees due to the change in Mach number and corresponding Cd value.

## Hornady 4 Degree of Freedom (4 DOF) Important Factors and Tips

## The following information is a guideline addressing input terms and important details and procedures to provide the most accurate trajectory predictions possible.

```
Choose Bullet
6 MM 105 BTHP V
```

Selection of projectile from drop down menu automatically loads mass, aerodynamic, and drag information specific to each projectile.

```
Muzzle Velocity
MIN 100 FPS MAX 4600FPS
```

    2600
    It is extremely important to have as accurate muzzle velocity data as possible. The Hornady 4 DOF drag predictions are based on Cd vs Mach values of each projectile. Ballistic Coefficient based calculators are based on Cd vs Mach values of a standard projectile. In both cases, Mach number accounts for a velocity value and its relationship to the speed of sound. If an inaccurate muzzle velocity is input, both the Hornady 4 DOF and BC based codes will use drag values for an erroneous Mach number. Although this error may be very small ( $2600 \mathrm{fps}=$ Mach 2.33 vs 2650 = Mach 2.37 ), it can have a measurable effect on point of impact at longer ranges when combined with the raw error in velocity alone. Muzzle velocity should be used, not instrumental velocity from a chronograph placed 10-20 ft. in front of the muzzle. Generally, a projectile will lose $10-15 \mathrm{fps}$ in the first 15 feet. See "Calculating Muzzle Velocity from Chronograph or Radar" on page 21.

## Zero Range <br> MIN OYDS MAX 2000YDS

Zero range should be based on a very accurate distance to the zero target as well as a very precise measurement of the average elevation of a group, at the given distance, relative to the point of aim. Simply shooting a 3 shot group at 100 or 200 yards, roughly measuring things and calling it good enough can produce large prediction errors in point of impact at long ranges. Table 4. represents errors in mean point of impact at 100 yards and the corresponding errors in point of impact with a trajectory ran for a true 100 yards zero. These values are not absolute and will vary in severity with projectile, velocity, atmospherics, and zero range.

| Mean Point of Impact at 100 YDS | Error at 1000 YDS |
| :---: | :---: |
| $.1^{\prime \prime}$ high or low | $1^{\prime \prime}$ |
| $.2^{\prime \prime}$ high or low | $2^{\prime \prime}$ |
| $.3^{\prime \prime}$ high or low | $3^{\prime \prime}$ |
| $.4^{\prime \prime}$ high or low | $4^{\prime \prime}$ |
| $.5^{\prime \prime}$ high or low | $5^{\prime \prime}$ |
| $.6^{\prime \prime}$ high or low | $6^{\prime \prime}$ |
| $.7^{\prime \prime}$ high or low | $7^{\prime \prime}$ |

Table 4. Impact Point Errors

It doesn't matter much where the actual impacts are relative to the point of aim in the vertical plane, only that it be a very accurate measurement of the average elevation point of impact relative to the point of aim. This provides the program with a precise point that all other results will be based on. This allows for the adjustment of the drag curve via the axial force form factor to the actual firearm/load for precise trajectory predictions at long ranges. Details of recommended zeroing practice can be found in the Additional Information section below on page 14.


Most ballistic programs available today compute an air density value based on the input atmospheric conditions. This air density is then assumed for all predictions of the projectile's flight. The Hornady 4 DOF creates an internal altitude based atmospheric table of conditions based on the provided inputs and accounts for the air density changes at the projectile's altitude during flight. Changes in air
density during the bullet's flight can become important if long range high angle shooting is conducted. A 1000 yard shot at a 30 degree angle results in a 1,500 foot change in altitude from launch to target. It is very important to know or have a very good estimate of the firing altitude in combination with atmospheric conditions.

```
Barrel Twist
MIN 7IN/REV MAX 20IN/REV
    7
```

Barrel twist inputs can be marked on the barrel or provided by the manufacturer. Correct barrel twist is critical for accurate gyroscopic stability (Sg), spin drift, and aerodynamic jump values. Always check the "Gyro" column of the outputs table first to ensure the projectile is stable at the muzzle, Sg greater than 1.4.
Bore Diameter
MIN $0.168 I N$ MAX 0.5 OIN
$6 \mathrm{~mm}, .243$

Bore dimensions are standardized and can be selected from the pull down menu based on bullet caliber. For non-standard bore dimensions, select the "other" option and manually input bore diameter.
Axial Force Form Factor
MIN 0.95 MAX 1.05
1.00

The axial force form factor option allows for a fixed percentage increase or decrease in drag (Cd value). This is not the same method employed when truing a BC on traditional ballistic programs. Variances in drag have been recorded between different guns, barrels and loads. By selecting a .95 Axial Force Form Factor the projectile drag values will be reduced by 5\%. Selecting a 1.06\% Axial Force Form Factor the drag will be increased by $5 \%$. This input allows the user to tweak the specific impact the barrel, load, etc. has on the drag of the projectile to provide the most accurate results possible.

When using the Axial Force Form Factor it is imperative that muzzle velocity, and zero range inputs are as accurate as possible. The axial force form factor should be changed from 1.00 as a last option after correctly measuring muzzle velocity and point of impact at zero range. Truing the muzzle velocity should be avoided and instead should be measured to avoid the use of erroneous Mach values during drag computations. For more detailed information please refer to the Hornady 4 DOF User Manual on page 1 and 2.

[^0]Wind angle inputs effect not only the values of wind drift but also the value and direction of aerodynamic jump. 0 headwind, 90 from the right, 180 tail wind, 270 from the left.

## Additional Information

## Recommended Zeroing Procedures

The importance of having a very detailed measurement of the relationship between point of impact (POI) and point of aim (POA) is described in the Important Factors and Tips section. Small errors in zeroing at relatively close ranges (100, 200 yards) can produce large errors between actual POI and predicted POI at longer ranges. Measurements during zeroing to within a minimum of a tenth of an inch are necessary and are easily achieved by the use of dial calipers. A ruler or other measurement method can be used with close attention to detail.

A limiting factor in achieving the perfect zero is the unit value of the sight adjustment. A . 25 MOA adjustment optic has a unit value of .26 " @ 100 yards, $.52^{\prime \prime} @ 200$ yards, and $.79^{\prime \prime}$ at 300 yard for 1 click. A 1/10 MRAD adjustment optic has a unit value of .36" @ 100 yards, .72 " @ 200 yards, and $1.1^{\prime \prime} @ 300$ yards for 1 click. The coarseness of these adjustment values allow for possible errors of one half of the inch value at their respective ranges. Example: If the center of a 100 yard group is $.13^{\prime \prime}$ above or below the point of aim, applying one click will result in the same error in the other direction for a .25 MOA adjustment. This possible prevention of the perfect zero is addressed in the procedures below.

## Procedures

1: With an accurately measured distance to the target, fire a 5 shot group without adjusting POA via turret adjustments or holding off.

2a: If individual bullet holes can be identified, measure the vertical distance between POA and the center of the POI of each shot. Average all of the values to find the vertical point of impact offset from the POA. See Figure 5.


Figure 5: Vertical Impact Point Measurement, Individual Holes Visible
2b: If individual bullet holes cannot be identified due to overlapping holes, measure the vertical size of the group and divide by 2 to find the center. Mark the location of the center of the group and measure the vertical location of the center from the POI. See Figure 6.


Figure 6: Vertical Impact Point Measurement, Individual Holes Not Visible


Figure 7: Close-up of Figure 6.

3: If the vertical distance between the POA and the POI is greater than the unit value of the sight being used, adjustments can be made to bring the difference closer to zero if desired, however this is not necessary for accurate outputs. Example: If the mean POI of the group is .6" high at 100 yards, everything
can be left alone if desired or the optic can be adjusted to bring the POI to within a minimum of a half of a click value as described in the introduction. The offset between POA and mean POI will be accounted for regardless of its value.

4: With a mean POI offset measured down to the nearest tenth of an inch, the Hornady 4 DOF should be provided with inputs with all information and conditions present during firing of the zero group.

5: If the mean POI is above the POA, set the zero range slightly beyond the zero target distance. Example: mean POI is $.6^{\prime \prime}$ high at 100 yds . Set zero distance of 4 DOF to 120 yds . If the mean POI is below the POA, set the zero range slightly closer than the zero target distance. Example: mean POI -.4" at 200 yds . Set zero distance of 4 DOF to 180 yds .

6: Run a trajectory and locate the drop column in inches. Find your zero range and note the height of the trajectory at your zero range. See Table 5.

Trajectory Results

| RANGE (YDS) | total COME UP ( ${ }^{(N)}$ | TOTAL WINDAGE (IN) | TRAJECTORY <br> (IN) | $\begin{aligned} & \text { AERODYNAMIC } \\ & \text { JUMP (IN) } \end{aligned}$ | $\begin{aligned} & \text { WIND } \\ & \text { DRIFT } \\ & \text { (IN) } \end{aligned}$ | $\begin{gathered} \text { SPIN } \\ \text { DRIET } \end{gathered}$ $(\mathbb{I N})$ | VELOCITY <br> (FPS) | ENERGY (FT-LB) | TOF (SEC) | GYRO |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | +1.5 | 0 | -1.50 | -0.00 | 0.00 | 0 | 2600 | 1577 | 0.000 | 2.08 |
| 10 | +1.07 | 0 | -1.07 | 0.00 | 0.00 | 0 | 2582 | 1554 | 0.012 | 2.10 |
| 20 | +0.7 | 0 | -0.70 | 0.00 | 0.00 | 0 | 2563 | 1533 | 0.023 | 2.13 |
| 30 | +0.38 | 0 | -0.38 | 0.00 | 0.00 | 0 | 2545 | 1511 | 0.035 | 2.15 |
| 40 | +0.11 | -0.01 | -0.11 | 0.00 | 0.00 | 0.01 | 2527 | 1489 | 0.047 | 2.17 |
| 50 | -0.1 | -0.01 | 0.10 | 0.00 | 0.00 | 0.01 | 2509 | 1468 | 0.059 | 2.19 |
| 60 | -0.26 | -0.02 | 0.26 | 0.00 | 0.00 | 0.02 | 2491 | 1447 | 0.071 | 2.21 |
| 70 | -0.36 | -0.02 | 0.36 | 0.00 | 0.00 | 0.02 | 2473 | 1427 | 0.083 | 2.23 |
| 80 | -0.41 | -0.03 | 0.41 | 0.00 | 0.00 | 0.03 | 2455 | 1406 | 0.095 | 2.25 |
| 90 | -0.39 | -0.04 | 0.39 | 0.00 | 0.00 | 0.04 | 2438 | 1386 | 0.107 | 2.27 |
| 100 | . 32 | -0.05 | 0.32 | 0.00 | 0.00 | 0.05 | 2420 | 1366 | 0.120 | 2.30 |
| 110 | -0.19 | -0.06 | 0.19 | 0.00 | 0.00 | 0.06 | 2403 | 1346 | 0.132 | 2.32 |
| 120 | -0 | -0.07 | 0.00 | 0.00 | 0.00 | 0.07 | 2385 | 1327 | 0.145 | 2.34 |
| 130 | +0.25 | -0.08 | -0.25 | 0.00 | 0.00 | 0.08 | 2368 | 1307 | 0.157 | 2.36 |
| 140 | +0.56 | -0.09 | -0.56 | 0.00 | 0.00 | 0.09 | 2350 | 1288 | 0.170 | 2.38 |
| 150 | +0.94 | -0.11 | -0.94 | 0.00 | 0.00 | 0.11 | 2333 | 1270 | 0.183 | 2.40 |
| 160 | +1.38 | -0.12 | -1.38 | 0.00 | 0.00 | 0.12 | 2316 | 1251 | 0.196 | 2.43 |
| 170 | +1.89 | -0.14 | -1.89 | 0.00 | 0.00 | 0.14 | 2299 | 1233 | 0.209 | 2.45 |
| 180 | +2.46 | -0.15 | -2.46 | 0.00 | 0.00 | 0.15 | 2282 | 1214 | 0.222 | 2.47 |
| 190 | +3.09 | -0.17 | -3.09 | 0.00 | 0.00 | 0.17 | 2265 | 1197 | 0.235 | 2.50 |
| 200 | +3.8 | -0.19 | -3.80 | 0.00 | 0.00 | 0.19 | 2248 | 1179 | 0.248 | 2.52 |

## Table 5: Reference Trajectory

7: Adjust zero distance of 4 DOF zero input and re-run the trajectory until the inch value at zero range matches mean POI measured at zero range.

| $\begin{aligned} & \text { RANGE } \\ & \text { (VDSE } \end{aligned}$ (YDS) | total COME UP (IN) | TOTAL WINDAGE (IN) | TRAJECTORY (IN) | AERODYNAMIC JUMP (IN) | $\underset{\text { DRIIT }}{\text { Wind }}$ <br> (iN) | $\begin{aligned} & \text { SPIN } \\ & \text { DRIFT } \\ & \text { (IN) } \end{aligned}$ | $\begin{aligned} & \text { VELOCITY } \\ & \text { (FPS) } \end{aligned}$ | ENERGY <br> (FT-LB) | TOF <br> (SEC) | GYRO |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | +1.5 | 0 | -1.50 | -0.00 | 0.00 | 0 | 2600 | 1577 | 0.000 | 2.08 |
| 10 | +1.05 | 0 | -1.05 | 0.00 | 0.00 | 0 | 2582 | 1554 | 0.012 | 2.10 |
| 20 | +0.65 | 0 | -0.65 | 0.00 | 0.00 | 0 | 2563 | 1533 | 0.023 | 2.13 |
| 30 | +0.3 | 0 | -0.30 | 0.00 | 0.00 | 0 | 2545 | 1511 | 0.035 | 2.15 |
| 40 | -0 | -0.01 | -0.00 | 0.00 | 0.00 | 0.01 | 2527 | 1489 | 0.047 | 2.17 |
| 50 | -0.24 | -0.01 | 0.24 | 0.00 | 0.00 | 0.01 | 2509 | 1468 | 0.059 | 2.19 |
| 60 | -0.42 | -0.02 | 0.42 | 0.00 | 0.00 | 0.02 | 2491 | 1447 | 0.071 | 2.21 |
| 70 | -0.55 | -0.02 | 0.55 | 0.00 | 0.00 | 0.02 | 2473 | 1427 | 0.083 | 2.23 |
| 80 | -0.63 | -0.03 | 0.63 | 0.00 | 0.00 | 0.03 | 2455 | 1406 | 0.095 | 2.25 |
| 90 | -0.64 | -0.04 | 0.64 | 0.00 | 0.00 | 0.04 | 2438 | 1386 | 0.107 | 2.27 |
| 100 | -0.6 | -0.05 | 0.60 | 0.00 | 0.00 | 0.05 | 2420 | 1366 | 0.120 | 2.30 |
| 110 | -0.49 | -0.06 | 0.49 | 0.00 | 0.00 | 0.06 | 2403 | 1346 | 0.132 | 2.32 |
| 120 | -0.33 | -0.07 | 0.33 | 0.00 | 0.00 | 0.07 | 2385 | 1327 | 0.145 | 2.34 |
| 130 | -0.11 | -0.08 | 0.11 | 0.00 | 0.00 | 0.08 | 2368 | 1307 | 0.157 | 2.36 |
| 140 | +0.18 | -0.09 | -0.18 | 0.00 | 0.00 | 0.09 | 2350 | 1288 | 0.170 | 2.38 |
| 150 | +0.53 | -0.11 | -0.53 | 0.00 | 0.00 | 0.11 | 2333 | 1270 | 0.183 | 2.40 |
| 160 | +0.94 | -0.12 | -0.94 | 0.00 | 0.00 | 0.12 | 2316 | 1251 | 0.196 | 2.43 |
| 170 | +1.42 | -0.14 | -1.42 | 0.00 | 0.00 | 0.14 | 2299 | 1233 | 0.209 | 2.45 |
| 180 | +1.96 | -0.15 | -1.96 | 0.00 | 0.00 | 0.15 | 2282 | 1214 | 0.222 | 2.47 |
| 190 | +2.57 | -0.17 | -2.57 | 0.00 | 0.00 | 0.17 | 2265 | 1197 | 0.235 | 2.50 |
| 200 | +3.25 | -0.19 | -3.25 | 0.00 | 0.00 | 0.19 | 2248 | 1179 | 0.248 | 2.52 |

Table 6: Adjusted Trajectory

This method of zeroing properly accounts for seemingly small differences in POI and POA during trajectory calculations. These seemingly small errors at zero range can cause large errors at longer ranges and should be accounted for to within a tenth of an inch minimum resolution. This method also allows the zero range to be precisely dialed in for mean POI that is within the adjustment value of the optic offering the user one more level of precision computation.

If multiple different loads will be used in a single rifle this same method can be used to compute zero's for each different load even though it may have a difference in POI in the vertical plane.

## Understanding Aerodynamic Jump

Aerodynamic jump can be a difficult concept to understand, but is an important factor of projectile trajectory to account for when attempting to make first round hits at long range in windy conditions. This explanation will provide visual and textual descriptions of aerodynamic jump to assist the shooter in understanding and utilize the concepts causing aerodynamic jump.

Figure 8 . is a representation of the pattern of the projectile's nose after exiting the muzzle under a no wind condition viewed from the base of the bullet as it travels down range, Alpha is pitch up or down, Beta to the right. The pitch and yaw resulting from muzzle exit and the gyroscopic properties of the spinning projectile cause it to precess around the flight path or velocity vector in an ever diminishing pattern. The pitch and yaw angle is continually getting smaller because the bullet is dynamically stable. As the projectile moves downrange, the yaw of repose causes the nose to point slightly to the right and slightly up as can be seen by the diminishing angles. The average y axis orientation of the projectile's nose is .000133 degrees above 0 and the average $x$ position of .003354 degrees right of 0 . This is the process often referred to as "going to sleep".


Figure 8. Projectile Pitch \& Yaw Angles No Crosswind

Figure 9. shows the same conditions as Figure 8. but with the addition of a 1 MPH wind at 90* (right to left). The introduction of a crosswind causes a dramatically different response from the projectile. The presence of a crosswind causes the precession cycles to have a much larger initial angle as well as an increased frequency of nutation. The average y axis orientation of the projectile's nose being . 000361 degrees above 0 and the average $x$ axis position of .00334 degrees right of 0 . The 1 MPH 90* wind has resulted in the projectile nose to point slightly higher than the no wind condition. This small angle of upward orientation of the projectile in relation to the velocity vector causes the upward jump seen when shooting in a 90* wind.


Figure 9. Projectile Pitch and Yaw Fired in a 1 MPH Cross Wind

Figure 10. is a combination of Figure 8, Figure 9. and a plot representing a $10 \mathrm{MPH}, 90$ degree crosswind. The small red area at the very center is a plot of Figure 6 to scale, for no wind. The area inside the green box is plot of Figure 7 To scale, for a 1 MPH crosswind. The large plot is the projectile response for the 10 mph crosswind. The influence of wind velocity is a large player in the projectile's behavior and the resulting aerodynamic jump values.


Figure 10. Combination of Figure 8. and Figure 9.

Table 7 is a comparison of average projectile $Y$ axis nose position at different wind velocities and directions. The 90 degree wind causes the bullet's nose to point slightly above the velocity vector as has been seen in Figure 8 and Figure 9. A 270 degree wind (left to right) has the opposite effect and causes the bullet nose to point downward, resulting in the downward jump angle when shooting in a 270 degree wind.


Table 7: Average Nose Position vs. Wind Speed \& Direction

Varying wind direction has a relatively small effect on aerodynamic jump values and the corresponding addition or subtraction from bullet drop values out to 1000 yds. Figure 11. shows aerodynamic jump values for a 10 mph headwind, tailwind, both full value wind directions, as well as the 4 half value wind directions. The positive values of the right to left direction winds are due to the bullet's nose up orientation due to the wind direction. The negative values for the left to right wind directions are due to the bullet's nose down orientation. Head and tailwinds have the least effect on vertical point of impact, and have long been known to be of little value out to 1000 yds. Half value winds have a greater effect than head or tail winds, but not to the full magnitude of a full value crosswind.

## Muzzle Jump 10 mph Wind



Figure 11. Aerodynamic Jump vs. Range \& Wind Direction, 100-1000 Yards

At very long ranges, the full value wind directions (90 and 270 degrees) remain fairly linear in their values. As the wind begins to approach the bullet at non-full value angles, in this example half value directions, the early linear behavior quickly accelerates in value. As the bullet continually bleeds velocity traveling downrange, the velocity of the assumed constant wind acting on the bullet becomes a growing percentage of the projectile's velocity. This is true for both half value wind directions as well as head and tail winds in this example. Due to the wind vector's alignment with the velocity vector for half value, head, and tailwinds, an axial component as well as a jump component are acting on the projectile.

The 225 degree wind direction coming from the left to right causes the bullet to nose down slightly giving it a downward aerodynamic jump that is seen in Figure 12. However, the dramatic loss of forward velocity
due to axial drag makes the velocity of the tailwind a higher percentage of the projectile's retained velocity once it is beyond 1000 yds . The tailwind acts to reduce the axial drag acting on the projectile compared to flight in still air. At high velocity, the $14.7 \mathrm{fps}(10 \mathrm{MPH})$ wind has very little effect on reducing the axial drag. Once velocity has bled from 2700 to 950 though, that same 14.7 fps wind velocity is of much greater value, percentagewise, when compared to forward bullet velocity. The effect of decreased axial drag from the tail wind is now greater than the aerodynamic jump component and overriding it enough to give the projectile an impact point above the 0 line beyond 1500 yards, as shown in Figure 8 and Figure 9.

The head and tailwind ( 0 and 180 degrees) that were of little value to 1000 yards have now become the highest values at 2000 yards. The head and tailwinds have no aerodynamic jump value as there is no crosswind component acting on the projectile. The increase and decrease in axial drag values of head and tailwinds become a very important consideration at extended ranges.


Figure 2: Vertical Aerodynamic Jump vs. Wind Direction, 1000-2000 Yards

Uphill or downhill angle shooting with a head or tailwind component introduces another interesting response from the projectile. During angle shooting, the velocity vector of the bullet is no longer parallel or near parallel to the head or tailwind wind velocity vector. Instead, as the angle of the shot is increased the velocity vector of the bullet becomes more perpendicular to the wind velocity vector with increased angle. This relationship can be viewed in the same way as a flat fire shot with a crosswind where the wind is acting at some degree of perpendicularity to the bullets velocity vector. In the example of the angle shot with the head and tailwind it is the same effect, but in the vertical plane instead of the horizontal. The corresponding aerodynamic jump value from the head or tailwind during an angled uphill or downhill shot will cause aerodynamic jump in the horizontal direction.

## Calculating Muzzle Velocity from Chronograph or Radar

The Hornady 4DOF requires the input of true muzzle velocities. If instrumental velocities are mistakenly used as muzzle velocities, errors will result. Instrumental velocities are gathered from a chronograph placed a certain distance in front of the muzzle, or from a Radar placed beside the barrel. In each case corrections need to be made to calculate true muzzle velocity.

## Chronograph

Chronographs are typically placed 5-20 feet in front of the muzzle when measuring velocity. The separation distance helps prevent muzzle blast from falsely triggering the chronograph sensors and providing bad readings. If a chronograph is used to gather velocity data, a detailed measurement should be taken from muzzle to the center point between the two sensors. Every attempt should be made to place the center of the chronograph at a whole yardage or meter value form the muzzle such as $3,4,5$, 6 yards or $3,4,5$, or 6 meters. Fractional distances such as 3.6 yards or 4.8 meters will make it much more difficult to calculate accurate muzzle velocities.

## Procedures

1: Place the center of the chronograph at a whole number yardage or meter value with enough separation between the muzzle and start sensor to not induce false triggers from muzzle blast.

2: Fire desired number of shots to record velocity.
3: Average recorded velocities.
4: Input all values present during the recording of velocities into the 4DOF inputs page. Set the output interval distance to 1 . Input a muzzle velocity that is slightly faster than the average instrumental velocity. Muzzle velocities will typically be 10-20 fps higher than the measured average instrumental velocity. General velocity loss guidelines can be found based on the BC range in table 8. Calculate the trajectory.

| Bullet G1 BC | Average FPS Lost at 15 Feet |
| :---: | :---: |
| $.400-.449$ | 11 |
| $.450-.500$ | 10 |
| $.501-.550$ | 9 |
| $.551-.600$ | 8 |
| $.601-.650$ | 8 |
| $.651-.700+$ | 7 |

Table 8. General Velocity Loss at 15 Feet

5: In the output table, locate the distance from the muzzle to the center of the chronograph. If velocity at this distance is below the recorded average velocity from the chronograph, the trajectory will need to be re-run with a higher muzzle velocity. If the velocity at the distance is above the recorded average velocity from the chronograph, the trajectory will need to be re ran with a lower muzzle velocity number.

6: Once the output table velocity at the chronograph distance matches the recorded average instrumental velocity from the chronograph, the muzzle velocity has been correctly calculated and should be used for further trajectory calculations.

## Radar

Radar is typically placed to the side of the barrel when measuring velocity. This lateral offset between the barrel and radar create a radial velocity early in the measurement that must be accounted for to determine accurate muzzle velocities. The projectile has to "fly into the beam". This will result in inaccurate velocity values early in the measurement if not accounted for. This is typically achieved by precise measurements between the radar head and barrel. If there is any doubt to the correction accuracy of radial velocity, the same method used for calculating a muzzle velocity via a chronograph can be used with a radar. If this method is chosen, a distance should be picked farther down range to minimize the effects of radial velocity. This distance will be dictated by the offset between the radar head and barrel, but in general by $30-50$ yards radial velocity effects have been mitigated.

## Procedures

1: Fire desired number of shots to record velocity.
2: Average recorded velocities.
3: Input all values present during the recording of velocities into the 4DOF inputs page. Set the output interval distance to 1 . Input a muzzle velocity that is reasonable for load and barrel length being used. Calculate the trajectory.

5: In the output table, locate the distance which has been chosen to remove possible radial velocity (30+ yds). If velocity at this distance is below the recorded average velocity from the radar, the trajectory will need to be re ran with a higher muzzle velocity. If the velocity at the distance is above the recorded average velocity from the radar, the trajectory will need to be re ran with a lower muzzle velocity number.

6: Once the output table velocity at the chronograph distance matches the recorded average instrumental velocity from the chronograph, the muzzle velocity has been correctly calculated and should be used for further trajectory calculations.

## Uphill and Downhill Shoot Angles

Traditional ballistic calculators offer relatively accurate predictions at shoot angles below 15 degrees at moderate distances. At angles greater than 15 degrees, or increased distances, errors from the flat fire calculations will become more pronounced. The Hornady 4 DOF correctly accounts for angled shooting based on a multitude of factors to include velocity vector alignment with gravity, changes in air density with increasing or decreasing projectile elevation, and the relationship between projectile elevation, line of sight angle, and target.

It is common practice to correct for an angled shot in the field by multiplying the range to target or the drop value at a given range by the cosine value of the angle being fired. At angles less than 10 degrees this method is relatively accurate, but will result in much larger errors as the angle and range increase. Actual drop values from the Hornady 4 DOF compared to cosine corrected values can be seen in Table 9.

| RANGE | ANGLE | 4 DOF <br> DROP <br> INCHES | COSINE <br> CORRECTED DROP <br> INCHES |
| :---: | :---: | :---: | :---: |
| 500 | 10 | -43.6 | -45.2 |
| 500 | -10 | -43.4 | -45.2 |
| 500 | 15 | -40.7 | -44.3 |
| 500 | -15 | -40.6 | -44.3 |
| 500 | 20 | -37.1 | -43.1 |
| 500 | -20 | -36.8 | -43.1 |
| 500 | 30 | -27.9 | -39.8 |
| 500 | -30 | -27.7 | -39.8 |


| RANGE | ANGLE | 4 DOF <br> DROP <br> INCHES | COSINE <br> CORRECTED DROP <br> INCHES |
| :---: | :---: | :---: | :---: |
| 1000 | 10 | -275.7 | -284.3 |
| 1000 | -10 | -274.3 | -284.3 |
| 1000 | 15 | -259.6 | -278.9 |
| 1000 | -15 | -257.7 | -278.9 |
| 1000 | 20 | -238.2 | -271.3 |
| 1000 | -20 | -235.9 | -271.3 |
| 1000 | 30 | -184.2 | -250.0 |
| 1000 | -30 | -181.7 | -250.0 |

Table 9. 4 DOF vs Cosine Correction Method

## Applying Spin Drift and Aerodynamic Jump

The Hornady 4 DOF output table provides values for spin drift that are independent of wind drift and aerodynamic jump independent of trajectory. Due to the changing nature of wind direction and speed, the values for spin drift and aerodynamic jump will need to be applied based on the judgement of the shooter. If the shooting conditions are consistent with the input wind speed and direction, the shooter solution located in the greyed Total Come Up and Total Windage columns do the calculations described below automatically. If wind conditions change from the input values and the shooter doesn't have time or ability to account for them by changing inputs, the individual variables can be applied at the shooters discretion based on the change.

## Spin Drift

Spin drift from a right hand twist barrel will always cause the bullet to drift to the right when viewed from behind the rifle. Values of spin drift are constant if velocity, twist rate, and atmospheric conditions are the same. Table 10 shows the 1000 yard spin drift value highlighted in green and the 10 mph wind value in red. If a right to left ( 90 degree) 10 mph wind is present, the spin drift value should be subtracted from the overall wind drift value. If a 10 mph left to right ( 270 degree) wind is present the spin drift value should be added to the wind drift value. The calculated value highlighted in blue can be seen automatically calculated in the shooter solution. The example output in Table 10 is calculated with a 90 degree wind direction.

Table 10: Applying Spin Drift
Trajectory Results

| RANGE (YDS) | TOTAL COME UP (IN) | tOTAL WINDAGE (IN) | TRAJECTORY <br> (IN) | AERODYNAMIC JUMP (IN) | $\begin{aligned} & \text { WIND } \\ & \text { DRIFT } \\ & \text { (N) } \end{aligned}$ | $\begin{aligned} & \text { SPIN } \\ & \text { DRIITT } \\ & \text { (IN) } \end{aligned}$ | VELOCITY (FPS) | ENERGY (FT-LB) | TOF (SEC) | GYRO |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | +1.5 | 0 | -1.50 | -0.00 | 0.00 | 0 | 2600 | 1577 | 0.000 | 2.08 |
| 100 | -0.34 | 0.69 | 0.00 | 0.34 | 0.74 | 0.05 | 2420 | 1366 | 0.120 | 2.30 |
| 200 | +3.76 | 2.88 | -4.44 | 0.68 | 3.07 | 0.19 | 2248 | 1179 | 0.248 | 2.52 |
| 300 | +14.75 | 6.71 | -15.77 | 1.02 | 7.16 | 0.45 | 2084 | 1013 | 0.387 | 2.78 |
| 400 | +33.76 | 12.36 | -35.11 | 1.35 | 13.21 | 0.85 | 1926 | 865 | 0.537 | 3.08 |
| 500 | +62.15 | 20.07 | -63.84 | 1.69 | 21.48 | 1.41 | 1774 | 734 | 0.699 | 3.39 |
| 600 | +101.6 | 30.08 | -103.63 | 2.03 | 32.23 | 2.15 | 1629 | 619 | 0.875 | 3.74 |
| 700 | +154.17 | 42.74 | -156.54 | 2.37 | 45.83 | 3.09 | 1489 | 517 | 1.068 | 4.15 |
| 800 | +222.46 | 58.42 | -225.16 | 2.70 | 62.70 | 4.28 | 1354 | 428 | 1.279 | 4.84 |
| 900 | +309.73 | 77.63 | -312.77 | 3.04 | 83.39 | 5.76 | 1225 | 350 | 1.512 | 5.58 |
| 1000 | +420.23 | $100.98$ | -423.60 | 3.37 | 108.5 | 7.54 | 1106 | 285 | 1.770 | 5.98 |

## WIND DIRECTION



## Aerodynamic Jump

Aerodynamic jump is applied in a very similar way as spin drift, just in the vertical direction.
Aerodynamic jump, like wind drift, has a direct correlation to wind speed. A 10 mph wind will produce double the aerodynamic jump of a 5 mph wind of the same direction. The shooter must determine if the jump is in the up or down direction. A wind blowing right to left ( 90 degrees) will cause the jump to be in the up direction and therefore decrease the amount of Total Come Up, and a left to right ( 270 degrees) will cause the bullet to jump in the downward direction increasing the Total Come Up. The example output in Table 11 is calculated with a 90 degree wind and illustrates the application of aerodynamic jump.

Table 11: Applying Aerodynamic Jump

Trajectory Results

| RANGE (YDS) | TOTAL COME UP (IN) | TOTAL WINDAGE (IN) | TRAJECTORY <br> (IN) | AERODYNAMIC JUMP (IN) | $\begin{aligned} & \text { WIND } \\ & \text { DRIFT } \\ & \text { (IN) } \end{aligned}$ | $\begin{aligned} & \text { SPIN } \\ & \text { DRIFT } \\ & (\mathbf{N}) \end{aligned}$ | velocity (FPS) | EnERGY (FT-LB) | $\begin{aligned} & \text { TOF } \\ & (\mathrm{SEC}) \end{aligned}$ | GYRO |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | +1.5 | 0 | -1.50 | -0.00 | 0.00 | 0 | 2600 | 1577 | 0.000 | 2.08 |
| 100 | -0.34 | 0.69 | 0.00 | 0.34 | 0.74 | 0.05 | 2420 | 1366 | 0.120 | 2.30 |
| 200 | +3.76 | 2.88 | -4.44 | 0.68 | 3.07 | 0.19 | 2248 | 1179 | 0.248 | 2.52 |
| 300 | +14.75 | 6.71 | -15.77 | 1.02 | 7.16 | 0.45 | 2084 | 1013 | 0.387 | 2.78 |
| 400 | +33.76 | 12.36 | -35.11 | 1.35 | 13.21 | 0.85 | 1926 | 865 | 0.537 | 3.08 |
| 500 | +62.15 | 20.07 | -63.84 | 1.69 | 21.48 | 1.41 | 1774 | 734 | 0.699 | 3.39 |
| 600 | +101.6 | 30.08 | -103.63 | 2.03 | 32.23 | 2.15 | 1629 | 619 | 0.875 | 3.74 |
| 700 | +154.17 | 42.74 | -156.54 | 2.37 | 45.83 | 3.09 | 1489 | 517 | 1.068 | 4.15 |
| 800 | +222.46 | 58.42 | -225.16 | 2.70 | 62.70 | 4.28 | 1354 | 428 | 1.279 | 4.84 |
| 900 | +309.73 | 77.63 | -312.77 | 3.04 | 83.39 | 5.76 | 1225 | 350 | 1.512 | 5.58 |
| 1000 | $(420.23)$ | 100.98 | -423.60 | (3.37) | 108.52 | 7.54 | 1106 | 285 | 1.770 | 5.98 |

WIND DIRECTION


## Index

Choose Bullet: A pulldown menu allowing you to select the projectile for trajectory analysis. Once a projectile is selected, its unique mass and aerodynamic characteristics are selected within the trajectory engine.

Units: A pulldown menu offering selection between standard and metric units. Once selected all input values and output values will be in the unit system selected.

Velocity: Muzzle velocity input.

Max Range: Input value for maximum range 4 DOF will calculate to.

Interval: Defines at what distance outputs will be calculated. Example: 25 yd interval will output trajectory data every 25 yds to the max range.

Zero Range: Range at which the bullet crosses the line of sight. Also called sight in range.

Sight Height: Distance from centerline of bore to centerline of sight. For scoped rifles this is the distance from the center of the bore to the center of the scope main tube.

Shooting Angle: Angular input for uphill or downhill shooting. - is downhill, + is uphill.

Wind Speed: Wind speed input used to compute wind deflection and aerodynamic jump from crosswind.

Wind Angle: Wind angle in relation to the bullet. 0* headwind, 90* from the right, 180* tail wind, 270* from the left.

Altitude: Height above sea level. Unlike other ballistics programs, altitude and pressure must both be input into the 4 DOF. Based on altitude and pressure value, a custom table is built within the program to properly model atmospherics.

Pressure: Uncorrected atmospheric pressure. Typically gathered from handheld weather station or pressure at altitude table. Do not use corrected pressure commonly used in weather forecasts. Pressure and altitude must both be input into the HTE to build the custom atmospheric model during calculations.

Temperature: Ambient air temperature.

Humidity: Relative humidity percentage.

Barrel Twist: Distance in inches or cm required for rifling to make one complete revolution in barrel. Can be found on barrel or from manufacturer. Critical input for gyroscopic stability, spin drift, and aerodynamic jump outputs.

Bore Diameter: Diameter of bore dimension of barrel. Groove dimension is commonly used as the reference to bullet caliber. Example: . 308 diameter bullets are designed for barrels with .308 groove dimension. Bore diameters are standard for caliber and can be found in table below. Custom bore diameters should be input if known, otherwise automatic standard dimension will be loaded upon bullet selection. Bore diameter input is critical for gyroscopic stability, spin drift, and aerodynamic jump outputs.

| CALIBER | STANDARD BORE DIMENSION |
| :---: | :---: |
| $22 \mathrm{CaI}, .223,5.56 \mathrm{~mm}$ | .219 |
| $6 \mathrm{~mm}, .243$ | .237 |
| $6.5 \mathrm{~mm}, .264$ | .256 |
| $7 \mathrm{~mm}, .284$ | .277 |
| $30 \mathrm{Cal}, .308$ | .300 |
| .338 | .330 |
| 50 Cal | .500 |


[^0]:    Wind Angle
    MIN $0^{\circ}$ MAX $359^{\circ}$

